Efficient Portfolio Management
Designing Optimal Strategies with Evolutionary Algorithms

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impa

Seminários

Métodos Matemáticos em Finanças

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Abstract

This work combines Markowitz–Sharpe and Black–Scholes models in such a way as to take advantage of the best of both worlds, by introducing a multi-asset continuous time framework for Monte Carlo simulations.

Abstract

Such a framework takes into account any costs, including taxation, and allows for shocks in both return and volatility, thus becoming a convenient background for the search of quasi-optimal investment strategies via genetic algorithms.

Abstract

This allows for an efficient portfolio management, which keeps the overall risk low due to a Markowitz diversification, while taking advantage of the market fluctuations perceived by the Black–Scholes model.

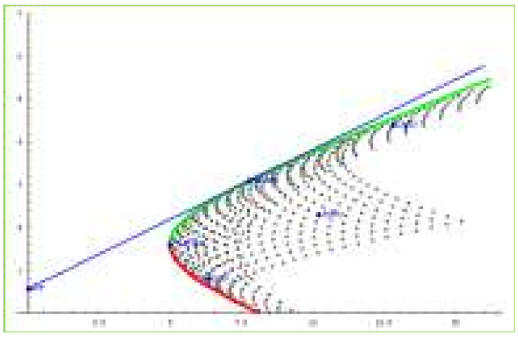
Abstract

Numerical results, based on real data from 2005 to 2009 for a “blue chips” portfolio in the BOVESPA stock exchange, show a definite improvement over a CAPM “buy-and-hold" pure strategy.

Timeline

- 1952 – H. Markowitz:
  Portfolio Theory
- 1964 – W. Sharpe:
  Capital Asset Pricing Model (CAPM)
- 1973 – F. Black & M. Scholes:
  Black–Scholes Analysis
- 1992 – R. Bauer & G. Liepins:
  Genetic Algorithms for Trading Strategies
- 1993 – D. Luenberger:
  Log-Optimal Model

Markowitz Model



Markowitz Model

The Good:

- Risk–Return Framework.
- Relevance of Diversification.
- Correlation Matrix.
- Optimization.
- Minimal Variance Portfolio.
- One Fund Theorem.
- Scientific–philosophical soundness.

Markowitz Model

The Bad:

- One Period Only.
- Buy–and–Hold Strategy.
- Constant Parameters.
- Computational Complexity.

Markowitz Model

The Ugly:

- Instability of the One Fund.
- Unreliable Expected Return Estimates.
- It Does Not Work at All on the Long Run.

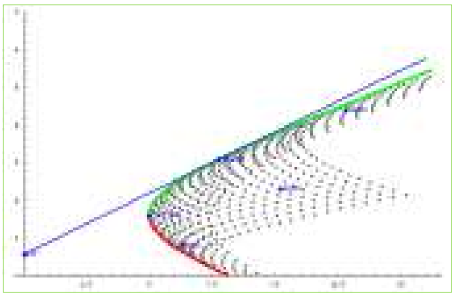
Capital Asset Pricing Model (CAPM)

Hypotheses:

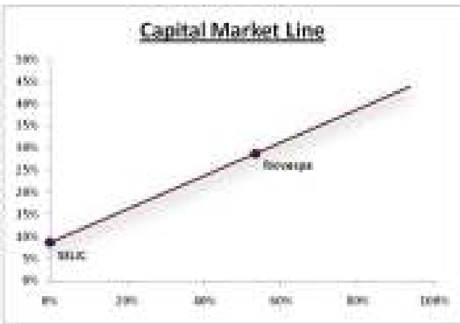
- All investors use Markowitz Model.
- All investors use the same risk–return estimates.
- All investors have access to unlimited credit at the risk–free interest rate.
- All investors pay no transaction costs or taxes.

Conclusion: All investors do make the same choices.

Capital Asset Pricing Model (CAPM)



Capital Asset Pricing Model (CAPM)



Capital Asset Pricing Model (CAPM)

The Good:

- Market Equilibrium.
- Market Portfolio.
- Capital Market Line.
- Market Index.
- Pricing Formula.
- Scientific–Philosophical Soundness.

Capital Asset Pricing Model (CAPM)

The Bad:

- Hard to Believe Hypotheses.
- Buy–and–Hold Strategy.
- Constant Parameters.
- No Asset Selection.

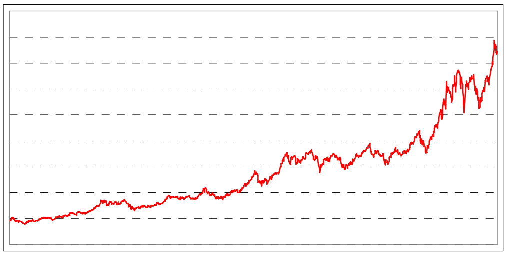
Capital Asset Pricing Model (CAPM)

The Ugly:

- Investment in All Assets.
- Instability of the Betas.
- It Works Only for Massive Investors.

Black–Scholes Analysis

The price of a stock as a Brownian motion:



Black–Scholes Analysis

Mathematically, the price follows an Ito process:

$$dS = \mu S\,dt + \sigma S \varepsilon \sqrt{dt}$$

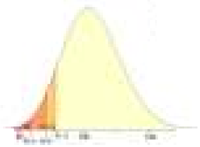
Expected Return

Volatility

Black–Scholes Analysis

Ito's lemma can then be applied to show, for instance, that stock prices have a log-normal distribution:

$$d(\ln S) = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma \varepsilon \sqrt{dt}$$



Black–Scholes Analysis

The combination of stochastic calculus with arbitrage theory leads to the famous differential equation for the price of a derivative:

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf$$

Black–Scholes Analysis

The Good:

- Continuous Time Framework.
- Arbitrage Theory.
- Irrelevance of Return for Derivative Pricing.
- Implied Volatility.
- The Greeks.
- Numerical methods for PDEs.
- Scientific–Philosophical Soundness.

Black–Scholes Analysis

The Bad:

- One Asset Only.
- Constant Parameters.
- Log-Normal Stock Prices Only.
- No Clue on Expected Returns.

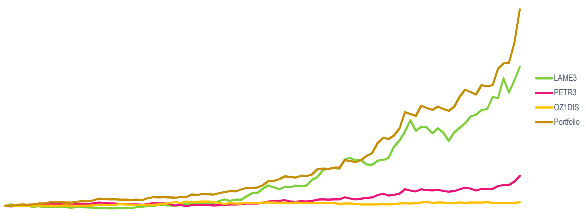
Black–Scholes Analysis

The Ugly:

- Volatility Smile.
- Term Structure of Volatility.
- It Does Not Work in Practice.

Genetic Algorithms for Strategies

A wise strategy can enhance a portfolio performance:



Genetic Algorithms for Strategies

Popular Trading Strategies:

- Market Timing:
  - Stops/Starts.
  - Technical Analysis.
- Security Picking:
  - Buy–and–Hold (including Markowitz and CAPM).
  - Volatility Pumping.
- Option Strategies.

Genetic Algorithms for Strategies

Bauer used genetic algorithms to find optimal trading strategies for market timing. His work used historical data only to set the parameters for taking advantage of market fluctuations of any one asset.

Genetic Algorithms for Strategies

The Good:

- Relevance of Trading Strategy.
- Considers Market Fluctuations.
- Optimization.
- Allows for Refinements.

Genetic Algorithms for Strategies

The Bad:

- One Asset Only.
- Usage of Historical Data Only.
- No Clue on Expected Returns or Implied Volatility.
- No Philosophical Soundness.

Genetic Algorithms for Strategies

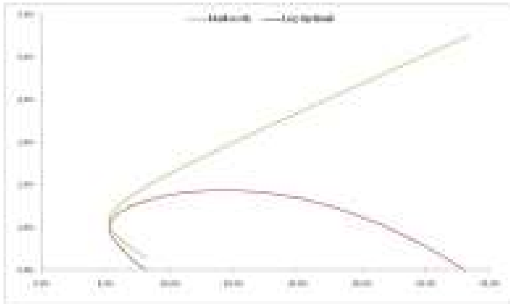
The Ugly:

- Ends Up in Technical Analysis Trading Rules.
- “Cristal Ball for the Past”.
- It Does Not Work when the Scenario Changes.

Log-Optimal Model

Luenberger combined results from
Markowitz model with Black–Scholes analysis
in order to improve that model for
long–term investors repeated use.

Log-Optimal Model



Log-Optimal Model

The Good:

- Risk–Return Improved Framework.
- Relevance of Diversification and Correlation Matrix.
- Optimization on the Long Run.
- Improved Log Utility Function.
- Minimal Variance Portfolio.
- One Fund Improved Theorem.
- Scientific–Philosophical Soundness.

Log-Optimal Model

The Bad:

- Buy–and–Hold Strategy.
- Constant Parameters.
- Computational Complexity.

Log-Optimal Model

The Ugly:

- Instability of the One Fund.
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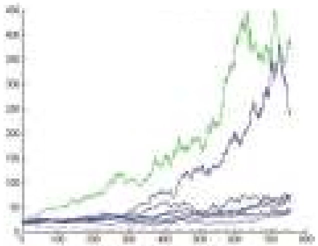
Monte Carlo Simulation

We improve Bauer's optimization by using Monte Carlo simulations based on the stochastic differential equations of Black–Scholes analysis. Historical data is used as an initial set of parameters, which do change over time as the simulation unfolds. This allows for changes or even shocks, both in expected return and volatility.

Monte Carlo Simulation

Such a framework allows us to take into account any costs, including taxation, custody, etc. Thus we get an extremely convenient background for the search of quasi-optimal investment strategies via genetic algorithms.

Monte Carlo Simulation



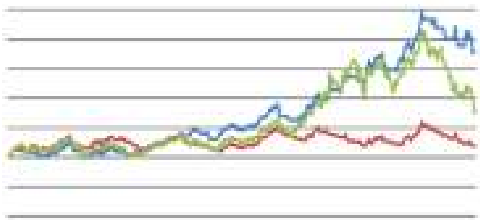
Randomly generated paths for PETR3.

The Genetic Algorithm

Steps of the Process:

- Initial Population of Trading Strategies.
- Selection Criterion (Utility Function).
- Reproduction Procedures:
  - Crossover (Sex).
  - Cloning.
  - Mutation.
- Successive Generations.

The Genetic Algorithm



Optimal strategy performance for PETR3.

The Multi-Dimensional Case

We run Monte Carlo simulations on several assets simultaneously. We generate paths for all assets that respect each asset's own parameters and have the appropriate correlation matrix.

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